Geometric Approach to Position Determination in Space: Advantages and Limitations

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Outline

- Kinematic POD with triple differences
- Data screening (CS detection)
- Orbit smoothing
- Achievable accuracy
- Summary
Kinematic POD

- **Advantages**
  - No force model error affects the solution
  - Fast (potential for near-real time)
  - Quality solution for good PDOP

- **Disadvantages**
  - No dynamics to compensate for weak geometry
  - No solution or weak solution for weak geometry
  - Requires correct coordinates for a starting epoch (forward solution only)
Triple Difference POD

- Triple difference kinematic precision orbit determination (POD)
  - OSU software GODIVA (1995): triple difference approach to GPS POD
  - OSU software P-KOD (Precision Kinematic Orbit Determination)
    - Extension of GODIVA to handle LEO (Low Earth Orbiter) POD in kinematic mode (2001)
  - UTX (Byun, S. H., 1998) LEO kinematic POD
Triple Difference POD

- Primary advantage: fast, no ambiguity fixing
- Disadvantage: epoch-to-epoch correlation (non-diagonal variance-covariance matrix)
  - Cholesky decomposition and decorrelation scheme
- Requires good approximated orbit to detect CS (large residuals)
- Equivalent to double difference with float ambiguities
P-KOD Data Processing: CHAMP

- 24 hour data sets processed
  - 65 IGS tracking stations
  - 30-s data sampling rate
  - Elevation cut off angle $0^\circ$ (CHAMP) and $10^\circ$ (stations)

- CS detection based on initial SNR prescreening, and triple difference residual analysis

- Normal matrix is accumulated until a singularity point is reached (too few observations or bad geometry)

- Initial epoch released (forward/backward filter)
P-KODE Processing Flowchart

IGS reference stations and GPS orbit data or OSU GODIVA

LEO observation data

Data prescreening
SNR analysis

Binary local data base

Station clock error estimation

Cycle slips detection

Construct triple phase differences

LEO orbit interpolation between epochs of observation

LEO POD Main Procedure

A priori values for LEO and station coordinates

Normal matrix

Forward/Backward Solution

Reduce normal matrix

Solution and update of the a priori values

Geodetic and Geoinformation Science
Example Results

- Good orbit approximation available to clean (remove) CS as large triple difference residuals
- One iteration allows for convergence
- Forward filtering (batch least squares)
- Backward filtering
- Average percentage of CS in the data
  - CHAMP: 5-6
  - Tracking stations: <0.5
### Distribution of Cycle Slips: 24 h Data Set, June 15, 2001

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total # of Epochs</td>
<td>2880</td>
</tr>
<tr>
<td># of Epochs with no C/S</td>
<td>2007</td>
</tr>
<tr>
<td># of Epochs with C/S</td>
<td>873</td>
</tr>
</tbody>
</table>

**Total No. of Observations:** 166495

- **Total = 9495 (5.7%)**
- **CHAMP:** 9226 (5.5%)
- **Stations (65):** 269 (0.2%)

- 97% of all CS
- 3% of all CS
Examples of Weak Geometry

<table>
<thead>
<tr>
<th>Epochs</th>
<th>RMS$_x$ [m]</th>
<th>RMS$_y$ [m]</th>
<th>RMS$_z$ [m]</th>
<th>RMS$_{3D}$ [m]</th>
<th>No. of Iterations</th>
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<tbody>
<tr>
<td>0683:0751 (068)</td>
<td>0.111</td>
<td>0.075</td>
<td>0.266</td>
<td>0.298</td>
<td>13</td>
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<tr>
<td>2396:2791 (395)</td>
<td>0.265</td>
<td>0.179</td>
<td>0.351</td>
<td>0.475</td>
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</table>

Geodetic and Geoinformation Science
# Statistics of Singularities

<table>
<thead>
<tr>
<th>Singularity</th>
<th>Epochs</th>
<th>Duration</th>
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<tbody>
<tr>
<td>1</td>
<td>0253 ~ 0255</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0678 ~ 0682</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>0752 ~ 0753</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1080 ~ 1080</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1314 ~ 1314</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1392 ~ 1392</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1904 ~ 1904</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2394 ~ 2395</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>2792 ~ 2792</td>
<td>1</td>
</tr>
<tr>
<td><strong>SUM</strong></td>
<td></td>
<td><strong>17</strong></td>
</tr>
</tbody>
</table>
Example Results

Forward filter solution

\[ RMS_x = 0.513 \text{ m} \]
\[ RMS_y = 0.865 \text{ m} \]
\[ RMS_z = 1.059 \text{ m} \]
\[ RMS_{3D} = 1.460 \text{ m} \]

Backward filter solution

\[ RMS_x = 0.079 \text{ m} \]
\[ RMS_y = 0.202 \text{ m} \]
\[ RMS_z = 0.155 \text{ m} \]
\[ RMS_{3D} = 0.266 \text{ m} \]
Example Results: October 3, 2001

- Data missing
  - Data gap: 56 epochs
  - Large clock error: 221 epochs
- Singularity due to weak geometry or insufficient data
  - 15 epochs
Example Results: October 3, 2001

Forward filter solution
RMS_x = 0.745 m
RMS_y = 1.029 m
RMS_z = 0.866 m
RMS_3D = 1.537 m

Backward filter solution
RMS_x = 0.173 m
RMS_y = 0.098 m
RMS_z = 0.154 m
RMS_3D = 0.252 m
SNR for CS Detection

- CS caused by low SNR due to bad ionospheric conditions, multipath, high receiver dynamics or low elevation angle
- Raw signal strength
- 3 types of SNR
  - S1, S2 : L1, L2 phase observations
  - SA : SNR for C/A channel (CHAMP ext.)
- Related to the elevation angle
SNR vs. Elevation
Cycle Slip Detection using SNR: CHAMP

Case (SA=120), (C/S using RSO=1657)

- Number of C/S combinations
- SNR (S2)
- # of C/S using SNR
- # of matched C/S (RSO vs. SNR)
## Cycle Slip Detection Using SNR: CHAMP

<table>
<thead>
<tr>
<th>SA</th>
<th>S2</th>
<th>TD</th>
<th>SNR</th>
<th>0 – 5 deg</th>
<th>5 – 10 deg</th>
<th>10 – 15 deg</th>
<th>15 – 20 deg</th>
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<tbody>
<tr>
<td>120</td>
<td>22</td>
<td>O</td>
<td>O</td>
<td>80</td>
<td>29</td>
<td>0</td>
<td>0</td>
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<tr>
<td></td>
<td></td>
<td>O</td>
<td>X</td>
<td>10</td>
<td>18</td>
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<td>1</td>
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<tr>
<td></td>
<td></td>
<td>X</td>
<td>O</td>
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<td>21</td>
<td>0</td>
<td>0</td>
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<tr>
<td>121</td>
<td>22</td>
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<td>X</td>
<td>7</td>
<td>17</td>
<td>1</td>
<td>1</td>
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<tr>
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<td></td>
<td>X</td>
<td>O</td>
<td>19</td>
<td>25</td>
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<tr>
<td>122</td>
<td>22</td>
<td>O</td>
<td>O</td>
<td>84</td>
<td>31</td>
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<td>X</td>
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<td>16</td>
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<td>1</td>
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<td>X</td>
<td>O</td>
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<td>27</td>
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<td>22</td>
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<td>O</td>
<td>86</td>
<td>31</td>
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<td>0</td>
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<tr>
<td></td>
<td></td>
<td>O</td>
<td>X</td>
<td>4</td>
<td>16</td>
<td>1</td>
<td>1</td>
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<tr>
<td></td>
<td></td>
<td>X</td>
<td>O</td>
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<td>28</td>
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<td>0</td>
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<td>124</td>
<td>22</td>
<td>O</td>
<td>O</td>
<td>87</td>
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<tr>
<td></td>
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<td>X</td>
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<td>15</td>
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<td>1</td>
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<td></td>
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<td>O</td>
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<td>3</td>
<td>14</td>
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<td>1</td>
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<tr>
<td></td>
<td></td>
<td>X</td>
<td>O</td>
<td>28</td>
<td>29</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- **X** – no C/S
- **O** – C/S
- **Total of 29 satellites tested over 500 epochs**
Cycle Slip Detection Using SNR: CHAMP

<table>
<thead>
<tr>
<th>SA</th>
<th>S2</th>
<th># of C/S</th>
<th># of matched C/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>22</td>
<td>1708 (5.0%)</td>
<td>1336 (80.6%)</td>
</tr>
<tr>
<td>121</td>
<td>22</td>
<td>1817 (5.3%)</td>
<td>1354 (81.7%)</td>
</tr>
<tr>
<td>122</td>
<td>22</td>
<td>1910 (5.7%)</td>
<td>1372 (82.8%)</td>
</tr>
<tr>
<td>123</td>
<td>22</td>
<td>1938 (5.6%)</td>
<td>1395 (84.2%)</td>
</tr>
<tr>
<td>124</td>
<td>22</td>
<td>1984 (5.8%)</td>
<td>1415 (85.4%)</td>
</tr>
<tr>
<td>125</td>
<td>22</td>
<td>2099 (6.1%)</td>
<td>1417 (85.5%)</td>
</tr>
</tbody>
</table>

- Number of C/S in TD residuals: 1657
- Total number of TD: 34374
- 500 epochs tested, all PRNs included
Cycle Slip Detection Using SNR: CHAMP

- OO – C/S detected by both methods (TD residual and SNR)
- OX – C/S indicated by TD residual only
- XO – C/S indicated by SNR only
Corresponding Orbit Solution: initial approximation good to ~ 5 m

Forward filter solution
- $\text{RMS}_x = 0.538$ m
- $\text{RMS}_y = 1.014$ m
- $\text{RMS}_z = 1.223$ m
- $\text{RMS}_{3D} = 1.677$ m

Backward filter solution
- $\text{RMS}_x = 0.120$ m
- $\text{RMS}_y = 0.245$ m
- $\text{RMS}_z = 0.193$ m
- $\text{RMS}_{3D} = 0.334$ m
Orbit Smoothing

- Guerra and Tapia (1974)
  - built-in FORTRAN function
  - works for the data with less than 25% error

- Moving averaging window
  - average of 20 data points

- Polynomial Fitting
  - 9th order
Orbit Smoothing

Direct Form II Transposed
- \( \text{RMS}_x = 0.092 \text{ m} \)
- \( \text{RMS}_y = 0.220 \text{ m} \)
- \( \text{RMS}_z = 0.154 \text{ m} \)
- \( \text{RMS}_{3D} = 0.283 \text{ m} \)

Polynomial fitting \((n=9)\)
- \( \text{RMS}_x = 0.099 \text{ m} \)
- \( \text{RMS}_y = 0.213 \text{ m} \)
- \( \text{RMS}_z = 0.154 \text{ m} \)
- \( \text{RMS}_{3D} = 0.280 \text{ m} \)
Summary

- Kinematic triple difference POD works well for good geometry
- Short processing time (less than 2 h, forward and backward, on 1.8 GHz Pentium processor)
- Problems with weak geometry
- CS cleaning is not easy (high dynamics, LEO in the middle of the ionospheric layer)
  - SNR plus orbit smoothing give promising results
  - More work needs to be done on SNR threshold selection
- Gaps in the solution – reduced dynamics needed for orbit continuity and balance between geometry and force model
ACKNOWLEDGEMENTS

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